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# How Beneficial Was the Great Moderation After All?

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## **Abstract**

In this paper I compute the welfare effect of the Great Moderation, using a consumption based asset pricing model. The Great Moderation is modelled according to the data properties of consumption and dividend growth, which display a reduction of their innovation-volatility and increased persistence. The theoretical model (a long-run risk model), calibrated to match average asset pricing variables in the data, is able to capture the two features of the Great Moderation, and it predicts a welfare loss caused by the Great Moderation (-0.9 percent), due mainly to the utility cost of a late uncertainty resolution.

JLE Classification: E32, G10.

Keywords: Great Moderation, welfare, long-run risk, asset pricing.

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# 1 Introduction

The Great Moderation has received an enormous amount of attention in the literature, much of it devoted to assessing a range of possible causal factors.<sup>1</sup> Relatively little research, however, has addressed whether the Great Moderation is important in terms of improving households' welfare. After all, how beneficial was the Great Moderation? In this paper I calculate the welfare change caused by the Great Moderation, and conclude that it is more than likely modest or even absent at all. A model-free approach suggests that the welfare gain of the Great Moderation is 0.11 percent in terms of household consumption. A model-driven approach implies a negative contribution of the Great Moderation on welfare (-0.9 percent in consumption equivalent terms). The main intuition behind the absence of a large gain coming from the Great Moderation relies on the increased persistence of consumption since the early eighties.

The procedure I follow to measure the welfare gain from the moderation is characterized by two important features. First, I show that computed welfare gains depend crucially on the assumed laws of motion of consumption before and after the Great Moderation. Therefore, a careful accounting of how both its variance and persistence have changed is required in order to assess changes in welfare. Additionally, because macroeconomic fluctuations are a source of risk for households, it seems natural that we should assess the gain from reducing this risk using a model which has empirically reasonable asset pricing implications<sup>2</sup>. After all, these observed prices are our best measures of how actual agents value risk. A second feature of my analysis, therefore, is to pay close attention to the asset pricing implications of the models used in my analysis.

The vast literature on the Great Moderation focuses mainly on the significant reduction in the variance of either the growth rates of macroeconomic variables, or of their business cycle components. I show, however, that the Great Moderation is characterized also by an increased persistence of both consumption and dividends<sup>3</sup>. Whereas the decline of the volatility of consumption and

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<sup>1</sup>Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Blanchard and Simon (2001) are among the pioneers of the literature on the Great Moderation. A survey of this literature can be found in Stock and Watson (2002).

<sup>2</sup>Alvarez and Jermann (2004) also analyse the relation between welfare cost of cycles and asset pricing in a "model-free" environment.

<sup>3</sup>Pancrazi (2013) and Pancrazi and Vukotic (2013) find similar results for consumption, investment, and output and for the Solow residual, respectively.

dividend innovation shocks has a stabilizing effect, the increased estimated persistence shifts the resolution of uncertainty from the short-run to the long-run. This finding is important because, depending on the preferences used to measure agents' welfare, we might actually expect to find no gain, or even a reduction in utility in the post-1983 period.

The following example serves to illustrate the importance of linking welfare calculations to asset prices. Lucas (1987) assessed the welfare cost of business cycle fluctuations using a simple representative-agent consumption-based asset pricing model with time separable constant relative risk aversion (CRRA) preferences. In his calibrated example the implied welfare gain of eliminating fluctuations is equivalent to a 0.01 percent increase in steady state consumption. Lucas' model, however, implies a negligible equity premium. If, instead, one calibrates the preference parameters, so that the equity premium in the model is 6 percent (its average value in the post-war period), the welfare gain from eliminating fluctuations rises to 7.5 percent of steady state consumption.

With these considerations in mind, I proceed as follows. To model the Great Moderation, I estimate a third order autoregressive model of real U.S. per capita consumption and dividends for both the pre-1983 and post-1983 periods. This model is sufficiently rich to capture the rich autoregressive structure of these two time-series in the two sub-samples. To measure welfare gains, I consider a long-run risk model as in Bansal-Yaron (2004). The model parameters are calibrated such that the model is able to match key asset pricing moments (first and second moments of risk-free return, average risky asset-return, average equity premium) across the two sub-samples. The ability of the long-run risk model to match the asset pricing moments is a consequence of the assumption of non-separable utility function (Epstein-Zin (1989) preference) and of the presence of a small and largely persistent component in the consumption and dividend process, i.e. the long-run risk.

The Great Moderation period differs from the pre-Great Moderation period in three dimensions. First, there is a reduction in the variance of the innovations of the consumption and dividend growth rate: I will label this effect as "innovation-effect". Second, there is an increased persistence of the consumption and dividend growth process: I will label this effect as "persistence-effect". Third, the asset pricing properties of the model in the Great Moderation suggests a small increase of

the relative magnitude of the long-run shock with respect to the consumption shock: I will label this effect as “long-run-risk-effect”. The main result of the paper is that these three effects jointly imply a reduction of welfare in the Great Moderation period with respect to the pre-1983 period, equal to -0.9 percent, in consumption equivalent terms.

Although this result might appear counterintuitive, my model-based welfare computation allows to derive an intuition for this finding by constructing counterfactual scenarios that can explain why the Great Moderation is welfare deteriorating in this model. If the Great Moderation were solely characterized by a reduction of the magnitude of the consumption and dividend shocks (innovation-effect), it would have had a sizeable welfare gain, equal to 1.6 percent in consumption equivalent. Intuitively, agents are risk averse and, *ceteris paribus*, they highly value the lower consumption variance. However, this scenario would also imply a drastic reduction of the mean equity premium, which was not observed in the data. The “persistence-effect” scenario captures the increased persistence of consumption and dividends without altering their unconditional variance and the long-run risk component. When the persistence of consumption and dividends increases, agents’ welfare is largely reduced (-6.6 percent), and the mean equity premium jumps to 25 percent. Intuitively, an agent endowed with Epstein-Zin (1989) preferences with intertemporal elasticity of substitution greater than one is adverse postponing uncertainty resolution. When the persistence of consumption and dividend increases, their volatility shifts from the short-run to the long-run, thus making the agent worse off. As a result, the agent would ask for a larger premium to hold a risky asset. A similar effect is induced by the “long-run-risk-effect”: since the higher relative increased variance of the long run risk induces also a larger long-run uncertainty, the agent assigns a negative welfare effect to it (-1.4 percent). Also in this case, he would ask for a larger equity premium to hold risky asset (12 percent).

In a nutshell, the proposed asset pricing framework suggests that the gain brought by the Great Moderation is rather small, if not absent at all. The reduction of the magnitude of the consumption and dividend innovation shocks implies an unambiguous welfare gain, as well as a reduction of the mean equity premium and its variance. But the Great Moderation period was not solely characterized by a reduction of consumption and dividends variance. The data, in fact, show

that the Great Moderation also features a larger portion of consumption and dividend variance concentrated in the long-run, caused by their increased persistence and by the larger relative long-run risk. These two properties are welfare deteriorating, since they delay the resolution of uncertainty from the investor’s point of view. As an additional support of the main finding, I show that a model-free approach as in Alvarez and Jermann (2004) also delivers a negligible welfare change from the Great Moderation, equal to 0.1 percent in consumption equivalent.

Finally, to assess whether other models predict a low gain from the Great Moderation, I also consider two alternative models used in the recent macro-finance literature that have been shown to successfully match key asset pricing facts: the non-linear habit model (Cochrane and Campbell (1999)), and the rare-disaster model (Rietz (1988) and Barro (2005, 2009) among others). Despite the attractive asset pricing qualities of Campbell and Cochrane (1999)’s habit model, the assumed peculiar form of non-linear relationship between habit stock and past consumption is problematic when computing the welfare change associated with a change in the law of motion of the endowment. In particular, the parameters of the law of motion of consumption implicitly affect the preference parameters that determine the sensitivity of the agent to consumption fluctuations. The lower the variance of consumption is, the more the habit stock responds to an endowment shock of a given magnitude<sup>4</sup>. This mechanism plays an important role in the model’s ability to match specific asset pricing facts, namely the first and second moments of the risk-free rate and the equity premium. However, it unfortunately obscures welfare calculations, because it is not possible to isolate the effects of changes in the exogenous process while holding the preference parameters fixed<sup>5</sup>. Considering the rare-disaster model, only a small fraction of the equity premium depends on the high-frequency properties of the consumption process, whereas it depends in large part on the probability and magnitude of rare disasters. If the Great Moderation is assumed to have left these features of the law of motion of consumption unchanged, there is little predicted change in the moments of financial variables, and only a very small welfare gain.

The paper is organized as follows. Section 2 presents the empirical analysis of the effect of

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<sup>4</sup>Ljungqvist and Uhlig (2009) also examines some related implications of the Campbell and Cochrane (1999) model for welfare calculations.

<sup>5</sup>When computing a “brute force” welfare changed of the moderation implied by the Campbell and Cochrane (1999) model, I obtain a welfare loss of 5 percent in consumption equivalent.

the Great Moderation on macroeconomic variables and asset pricing. Section 3 illustrates the relationship between welfare cost, asset prices, and law of motion of consumption. Section 4 presents the asset prices model and its calibration. Section 5 illustrates the computed welfare costs of the Great Moderation. Section 6 presents alternative models. Section 7 concludes with some remarks.

## 2 Great Moderation: Stylized Facts on Macroeconomic Variables and Asset Pricing

The extensive literature on the Great Moderation has focused mainly on the decline of the volatility of macroeconomic variables<sup>6</sup>. In this section I extend the analysis of the Great Moderation in two directions. First, I analyze how both variance and persistence of consumption and dividends has been affected by the Great Moderation period. Second, I investigate whether the Great Moderation has affected any of the key moments of some financial variables, such as average values and variance of the risk-free rate, the equity premium, and the Sharpe ratio.

### 2.1 Macroeconomic Variables

Consider the following two U.S. macroeconomic variables: real per capita aggregate consumption, measured as non-durable goods plus services, and real dividends<sup>7</sup>. The dataset includes observations from the period 1947Q1-2007Q4. I will refer to the sub-sample 1947Q1-1982Q4 as Sample 1 (the period before the Great Moderation), and the sub-sample 1983Q1-2007Q4 as Sample 2 (the period of the Great Moderation)<sup>8</sup>. The choice of 1983 as the break date for the beginning of the Great Moderation is in line with the large literature on this topic (see Stock and Watson (2002)). In this paper, I will mainly focus the empirical analysis on the growth rate of consumption and

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<sup>6</sup>See Stock and Watson (2002) for a review.

<sup>7</sup>Sources: consumption is obtained from NIPA, dividends are extracted from Robert Shiller's online database. <http://www.econ.yale.edu/~shiller/data.htm>

<sup>8</sup>The decision to include observation up to 2007Q4, thus excluding the recent crisis, is driven by the choice to minimize the variance of the macroeconomic variables in the second sub-sample, thus giving the Great Moderation the best chance to have a large welfare impact. Nevertheless, the stylized facts about the cyclical properties of macroeconomic real variables presented in this section are robust to including the more recent observations.



dividends. The underlying assumption that the two processes are characterized by an unit root is vastly common in the literature and it is consistent with the theoretical model studied in this paper<sup>9</sup>.

Table 1 presents some descriptive statistics of the time-series properties of consumption and dividend growth rate in the two sub-samples. Few important features are worth noticing. First, the mean of consumption and dividend growth is relatively stable across the sample. A Chow (1960) test fail to reject the hypothesis that the mean of consumption and dividend growth experienced a break during the Great Moderation<sup>10</sup>. This is an important result because I can abstract from changes in the level of consumption and dividends when computing the welfare gain of the Great Moderation. Second, the variance of both consumption and dividend growth has significantly declined during the Great Moderation, as expected. Finally, the correlation between consumption and dividend growth is rather low in the whole sample, in particular in the Great Moderation period.

Whereas the literature has largely documented the effect of the Great Moderation on the variance of macroeconomic variables, very little attention has been focused on its effect on the persistence<sup>11</sup>. In what follows I analyze the changes in persistence of the two variables of interest in the two sub-samples. The first important step for this analysis is realizing that the time-series of dividend and consumption growth are both characterized by a rich autoregressive structure. In fact, econometric information criteria suggest that the two series are well described by a third order autoregressive process, as:

$$y_t = c + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + \sigma_\epsilon \epsilon_t \quad \epsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1), \quad (1)$$

where  $y_t$  indicates the variable of interest (consumption or dividend growth).

Table 2 displays the values of the estimated parameters for consumption and dividend growth in

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<sup>9</sup>The results presented in this section are robust to the assumption of the presence of a deterministic trend in consumption, as showed in Pancrazi (2013).

<sup>10</sup>The Wald statistics for the Chow (1960) test for the null hypothesis of constant mean of consumption and dividend growth in the two subsamples are 0.01 and 0.15 respectively. The critical value at a 0.10 significance level is 2.72.

<sup>11</sup>Exceptions are Pancrazi (2013), and Pancrazi and Vukotic (2013)

the whole sample, in Sample 1, and in Sample 2, along with their standard errors. The important feature of the estimation is that in the three samples considered there is a large number of significant autoregressive parameters of order two and three.

Have the properties of consumption and dividends changed during the Great Moderation period? To answer this question I compute two statistics describing the process in (1): its persistence, measured as the largest root of the autoregressive polynomial, and the standard deviation of the innovation,  $\sigma_\epsilon$ . Table 3 displays these estimated parameters for consumption and dividend growth in the three samples, along with the 90 percent confidence band computed via bootstrapping. The main results are the following. First, as vastly documented in the literature, the variance of the innovation of consumption and dividends has largely declined during the Great Moderation period<sup>12</sup>. Second, the persistence of consumption and dividends has increased in the Great Moderation period. This is a novel result in the literature, and it is consistent with the finding of Pancrazi (2013) for consumption, investment and output, and of Pancrazi and Vukotic (2013) for the Solow's residuals. As showed below, the properties of consumption and dividends are a crucial determinant for the realized asset pricing moments and for welfare: therefore, in this paper I will study a model that can both capture the declining variance of the innovations of consumption and dividends, and their increased persistence, and that can isolate their effects on welfare.

## 2.2 Asset Prices

In contrast to the vast literature on the stabilization of macroeconomic variables during the Great Moderation, relatively little attention has been paid to changes in the behavior of financial variables<sup>13</sup>. In this section I analyze some key moments of asset prices before and during the Great Moderation, to assess whether the reduction in the volatility of macroeconomic variables coincides with changes in the moments of financial variables. In particular, I first consider three time series: the real annualized return of a risk-free asset, measured as the return of 3-month Treasury bills,

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<sup>12</sup>The underlying assumption in this section is that the Great Moderation is an absorbing state. In Lettau, Ludvigson and Wachter (2008) the mean and volatility of the consumption process is modeled as a Markov process. In order to give the Great Moderation the best chance to imply the highest welfare gain as possible, in this paper I abstract from welfare effect of the regime uncertainty, thus focusing only on the welfare effects of the observed properties of consumption during the Great Moderation period.

<sup>13</sup>Lettau, Ludvigson, and Wachter (2008) analyze the increase of the price-dividend ratio in the 1990s.

the annual real return of equity, measured using the value-weighted market return defined by Fama and French<sup>14</sup>, and the risk premium, measured as the difference between the risk-free return and the equity return.

Table 4 shows the first and second moments of these variables in the two sub-samples. The average returns of the assets rose during the Great Moderation, by 2.3 percentage points for equities, and 1.6 percentage points for risk assets. As a result, the mean of the equity risk-premium grew only slightly in the post-1983 period, by 0.7 percentage points. However, a Chow (1960) test statistic (0.31) suggest that this small increase in the average risk premium is not statistically significant. Thus, I infer that the risk-premium did not change as a result of the Great Moderation. In addition, also the equity return volatility, the risk-premium volatility and the Sharpe ratio are approximately unchanged across the two sub-samples.

### 3 Welfare and Asset Pricing

Lucas (1987) concludes that the welfare gain from eliminating business cycle fluctuations is negligible. In this section I demonstrate, however, that welfare calculations depend on two important features of a model: the specification of the exogenous consumption process and the asset pricing implications of the model structure.

#### 3.1 Revisiting Lucas' Calculation

Lucas (1987) finds that the cost of business cycles is extremely low; an agent would agree to give up less than 0.04 percent of his consumption to avoid them entirely. However, to compute this cost Lucas (1987) uses two crucial assumptions: the logarithm of consumption is specified as an i.i.d. process around a linear trend, and CRRA utility is calibrated with a small coefficient of risk aversion. In this section I show that departures from these assumptions greatly affect the computed welfare cost. Specifically, I compute the welfare cost using the same CRRA utility specification as in Lucas, but I adopt the autoregressive process for consumption growth specified as in Mehra and Prescott (1985), which is fit to the 1889-1978 sample of U.S. data. I also calibrate the preference

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<sup>14</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

parameters so that the model matches the average equity premium and average risk-free rate in the Mehra and Prescott data set (1985).

To illustrate the sensitivity of welfare calculations to the model specification, in the first step, I assume that the preference parameters and the consumption process are specified as in Lucas (1987), i.e. the discount factor  $\beta$  is equal to 0.95, the coefficient of risk aversion is equal to 2, and the logarithm of consumption is i.i.d. around a linear trend, i.e.

$$\log(C_t) = gt + z_t \quad z_t \stackrel{iid}{\sim} N(0, \sigma_z^2).$$

Following Lucas'(1987) calibration, the mean growth rate of consumption,  $g$ , is set equal to 0.03 and the standard deviation of the stationary component,  $\sigma$ , is set equal to 0.013. As the first column of Table 5 shows, the model predicts a negligible welfare cost from eliminating the fluctuations equal to 0.017 percent. Moreover, the model is not able to predict a significant equity premium, as Mehra and Prescott (1985) pointed out.

Similar results can be obtained maintaining the assumption of a CRRA utility function, but assuming that the growth rate of consumption is an i.i.d. normal random variable with mean  $\mu$  and standard deviation  $\sigma$ , i.e.:

$$\log(C_t) = \log(C_{t-1}) + \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(\mu, \sigma^2).$$

Calibrating  $\mu = 0.03$  and  $\sigma = 0.013$  as estimated in the post-war period, the second column of Table 5 shows that the model predicts an equity premium close to zero, and a low welfare cost from eliminating the fluctuations equal to 0.1 percent in consumption compensation. In both the trend stationary and difference stationary specifications of the consumption process, the model prediction of a low equity premium is associated with a low welfare cost of the fluctuations.

The link between asset pricing and welfare cost can be defined analytically in a basic consumption-based asset pricing model with time-separable CRRA utility, like the proposed by Lucas (1978, 1987) and explicitly described in Appendix A.

As in Lucas, I first assume that the logarithm of consumption,  $c_t = \log(C_t)$ , is an i.i.d. process

around a linear trend, i.e.:

$$C_t = (1 + \mu)^t e^{z_t - \frac{1}{2}\sigma_z^2} \quad z_t \sim N(0, \sigma_z^2).$$

In this case, a first order approximation of the equilibrium implies that the welfare gain from eliminating fluctuations, expressed in consumption compensation terms, is:

$$\lambda = \frac{1}{2}\gamma\sigma_z^2, \tag{2}$$

and the approximated expected value of the equity premium is

$$E(R^{EP}) = \{\beta^{-1}(1 + \mu)^{-\alpha} - 1 + \gamma\}(1 + \mu)\gamma\sigma_z^2. \tag{3}$$

where  $\alpha = 1 - \gamma$ . Given equation (2) and equation (3), it is evident that the model prediction about the equity premium is tightly related to the welfare cost. In fact, the two variables are proportional since:

$$\lambda = \frac{1}{2} \{\beta^{-1}(1 + \mu)^{-\alpha} - 1 + \gamma\}^{-1} (1 + \mu)^{-1} E(R^{EP}).$$

Moreover, notice that both the equity premium and the welfare cost are tied to the coefficient of risk aversion,  $\gamma$ . Also, notice that, when  $\gamma \geq 1$ , a decline of the variance of the consumption-innovation,  $\sigma_z^2$ , implies a reduction of the expected equity premium and of the welfare gain.

### 3.2 Mehra and Prescott's Calibration

In the second step, I show that the specification of the consumption process affects the welfare computation. I follow Mehra and Prescott (1985), modeling the exogenous process as a first order autoregressive process for consumption growth:

$$\Delta \log(C_t) = \mu(1 - \rho) + \rho \Delta \log(C_{t-1}) + \sigma \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(0, 1),$$

where the mean  $\mu$  is calibrated to be 0.0179, the autoregressive coefficient  $\rho$  is calibrated to  $-0.139$ , and the standard deviation of the error term  $\sigma$  is calibrated to 0.0347. This model best fits the aggregate consumption data observed in Mehra and Prescott's (1985) sample period, 1889-1978. Using the same preference parameters as in the previous step ( $\beta = 0.95$   $\gamma = 2$ ), the welfare cost of business cycles is now 0.65 percent, 30 times larger than Lucas' (1987) estimate. Obstfeld (1994) reaches similar conclusions: under the unit-root assumption, innovations in growth have cumulative effects, which greatly affect welfare. However, the cost of the business cycle is still modest and, as Table 5 displays, the equity premium predicted by the model is still small.

Finally, in the third step I maintain the assumption of the autoregressive process for consumption growth, but I calibrate the preference parameters such that the model predicts a risk-premium of 6 percent and a risk-free return of close to 1 percent, the average values observed in Mehra and Prescott's (1985) sample. Table 5 shows that a coefficient of risk-aversion equal to 17 and a discount parameter greater than unity are able to generate asset returns whose first moments reasonably match the data<sup>15</sup>. In this scenario, the welfare cost of the fluctuations is large, about 7.4 percent. This result suggests that the welfare cost of business cycle fluctuations implied by a model is tightly related to the ability of that model to generate a large price for risk.

However, as shown in the previous section, none of these calculations is appropriate for thinking about the effects of the Great Moderation because it did not lead solely to a decline in of the consumption innovation volatility, but also to an increased consumption persistence. Therefore, a basic exercise in which consumption growth is the exogenous stochastic process, and the variance of its innovation declines, is completely silent about the effects of a changing autocorrelation structure on welfare and asset pricing.

In this paper I propose a particular solution to this problem, introducing a model in which the consumption process is flexible enough to capture the observed changing behavior of consumption (and dividends), and in which preferences are such that the model can match the key moments of asset prices in the post-war period. With this model I compute the effect of the Great Moderation on welfare, and conclude that the gain implied by the Great Moderation is rather small, even though the model predicts a large equity premium and a small risk-free rate.

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<sup>15</sup>Kocherlakota (1990) obtains similar results in an analogous exercise.

## 4 An Asset Pricing Framework

The previous sections have showed two important results. First, the Great Moderation has affected both the variance and the autocorrelation structure of consumption and dividends. Second, the welfare calculation implied by macroeconomic models are severely sensitive to their specification and, in particular, to their asset pricing implications. Therefore, in this section I introduce a model which is able to match some of the basic asset pricing moments, namely mean and variance of the risk-free rate, the equity return, and the Sharpe ratio, and that is able to capture the dynamics of consumption and dividends.

### 4.1 The Long-Run Risk Model

The model is similar to the asset pricing model with risks for the long-run as proposed by Bansal and Yaron (2004). The choice of using this model is motivated by several factors: it is a one of the few consumption based asset pricing models in the literature that can capture both the first and second moments of asset pricing, it allows to consider both the dynamics of consumption and dividends, and, finally, it is suitable for welfare calculations, as showed by Croce (2012)<sup>16</sup>. In this section, I consider two main departures from Bansal and Yaron(2004)'s benchmark model. First, I abstract from stochastic time-varying probability of the exogenous processes, and, second, I characterize the exogenous dynamics of consumption and dividend growth with a third-order autoregressive process, consistently with the empirical evidence reported in Section 2.

The representative agent has Epstein and Zin (1989) preferences that take the following form:

$$U_t = \left[ (1 - \sigma) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t [U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where  $0 < \delta < 1$  and  $\delta^{-1} - 1$  is the rate of time preference. Let  $\theta = \frac{1-\gamma}{1-1/\Psi}$ , where  $\gamma \geq 0$  is the risk aversion parameter, and  $\Psi \geq 0$  is the intertemporal elasticity of substitution. Notice that this preference specification allows to separate the risk aversion parameter from the intertemporal elasticity of substitution, a key feature for the model to be able to match asset pricing data.

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<sup>16</sup>Tallarini (2000) studies the welfare implications implied by Epstein-Zin (1989,1991) preferences.

There are  $N$  assets in this economy, whose one period ahead return is denoted as  $R_{j,t+1}$ ,  $\forall j = 1, \dots, N$ . The share of investment in asset  $j$  is denoted as  $\omega_{j,t}$ . Hence, there are  $N - 1$  linearly independent elements in  $\omega_t = [\omega_{1,t} \dots \omega_{N,t}]'$ , since the portfolio must satisfy:

$$\sum_{j=1}^N \omega_{j,t} = 1, \quad \forall t.$$

The representative agent is endowed with an initial stock of the consumption asset  $A_0$  that can be either consumed or invest in the  $N$  assets in the competitive market. Therefore, her wealth evolves according to:

$$A_{t+1} = (A_t - C_t) \omega_t' R_t.$$

Following Epstein and Zin (1991), the first order condition that prices an asset that entitles a claim on a stream of consumption good implies:

$$v_{c,t} = E_t \left[ \delta^\theta e^{\left[-\frac{\theta}{\Psi} \Delta c_{t+1} + (\theta-1)r_{c,t+1}\right]} (1 + v_{c,t+1}) e^{\Delta c_{t+1}} \right], \quad (4)$$

where  $\Delta c_t$  is the growth rate of consumption (exogenously specified below),  $r_{c,t+1}$  is the logarithm of the gross return of a consumption claim, and  $v_{c,t}$  is the price-consumption ratio, i.e.:

$$\begin{aligned} \Delta c_{t+1} &= \log \left( \frac{C_{t+1}}{C_t} \right), \\ r_{c,t+1} &= \log (R_{c,t+1}), \\ R_{c,t+1} &= \frac{(1 + v_{c,t+1}) e^{\Delta c_{t+1}}}{v_{c,t}}, \\ v_{c,t} &= \frac{P_{c,t}}{C_t}. \end{aligned}$$

Equivalently, the first order condition to price an asset that entitles to a dividend  $D_t$  in each period implies:

$$v_{d,t} = E_t \left[ \delta^\theta e^{\left[-\frac{\theta}{\Psi} \Delta d_{t+1} + (\theta-1)r_{d,t+1}\right]} (1 + v_{d,t+1}) e^{\Delta d_{t+1}} \right], \quad (5)$$

where  $\Delta d_t$  is the growth rate of dividends (exogenously specified below), and  $v_{d,t}$  is the price-



dividend ratio, i.e.:

$$\begin{aligned}\Delta d_{t+1} &= \log\left(\frac{D_{t+1}}{D_t}\right), \\ v_{d,t} &= \frac{P_{d,t}}{D_t}.\end{aligned}$$

Finally, we close the model specifying exogenous processes for consumption and dividend growth. I assume they are described by a third order autoregressive process in order to capture the rich autocorrelation structure of estimated for the consumption and dividend growth time-series. Thus:

$$\Delta c_{t+1} = \mu_c + \theta_1^c \Delta c_t + \theta_2^c \Delta c_{t-1} + \theta_3^c \Delta c_{t-2} + x_t + \sigma_c \epsilon_{t+1}^c \quad (6)$$

$$\Delta d_{t+1} = \mu_d + \theta_1^d \Delta d_t + \theta_2^d \Delta d_{t-1} + \theta_3^d \Delta d_{t-2} + \lambda_d x_t + \sigma_d \epsilon_{t+1}^d \quad (7)$$

$$x_{t+1} = \rho_x x_t + \sigma_x \sigma_c \epsilon_{t+1}^x \quad (8)$$

where the shocks  $\epsilon_t^c$ ,  $\epsilon_t^d$ ,  $\epsilon_t^x$  are *i.i.d.* normally distributed with mean zero and variance one;  $x_t$  captures the long-run risk component of consumption and dividend growth. Notice that  $\sigma_x$  scales the standard deviation of the long-run risk component to be a fraction of the standard deviation of the consumption growth innovation, in line with Croce (2012). Importantly, Bansal and Yaron (2004) pointed out that the long-run risk process,  $x_t$ , should be described by a high and positive persistence  $\rho_x$  and by a low variance relative to the consumption growth innovation,  $\sigma_x$ . Finally, the parameter  $\lambda_d$  determines the joint comovement of dividends and consumption. The economy is fully described by equations (4) – (8). I derive the solution of the model by using a third-order approximation of the equilibrium conditions around the steady state.

## 4.2 Calibration

In order to calibrate the model I propose the following strategy: I consider first the estimated autoregressive parameters of the consumption process,  $[\theta_1^c, \theta_2^c, \theta_3^c]$ , and dividend process,  $[\theta_1^d, \theta_2^d, \theta_3^d]$  as reported in Table 2, for the whole sample 1947:1-2007:4. I then calibrate the two parameters

of the long-run risk process,  $x_t$ , in line with the literature on the long-run risk. In particular, I fix the persistence of  $x_t$  to be  $\rho_x = 0.98$  as in Bansal and Yaron (2004) and  $\sigma_x$  to be a small percentage (3 percent) of the standard deviation of consumption  $\sigma_c$ . The high-persistence and the small standard deviation of the error term of the long-run risk process are two crucial features of this model, which make this process not able to be identified directly from consumption data. Finally, I calibrate the remaining parameters of the exogenous processes  $[\mu_c, \mu_d, \sigma_c, \sigma_d]$  to match the mean and standard deviation of the consumption and dividend growth, and  $\lambda_d$  to match their correlation in the whole sample 1947:1-2007:4.

Three are the preference parameters to be calibrated. The value for the intertemporal elasticity of substitution  $\Psi$  is taken from Bansal and Yaron (2004) and it is equal to 1.5, since they provide evidence of that parameter being greater than one. The discount factor is calibrated at  $\delta = 0.998$  as in Bansal and Yaron (2004) and Croce (2012). Finally, the value of the coefficient of risk aversion  $\gamma$  is chosen to match the Sharpe ratio implied by the model. The implied value is  $\gamma = 12$ , in line with the magnitude used by Croce (2012).

The first column of Table 6 reports the complete calibration for the whole sample, and Table 7 reports the performance of the model on matching the properties of consumption and dividend growth in the whole sample, as well as the asset price data moments. Notice that by matching almost exactly the properties of consumption and dividends, the model is able to replicate remarkably well the first and second moments of asset prices. Whereas the Sharpe ratio and the mean equity premium are targeted moments to pin down  $\gamma$  and  $\sigma_x$ , the magnitude of the mean equity rate and risk-free rate and their variances are not targeted moments: even though the model-implied risk-free rate is slightly larger than the one observed in the data, I can claim that the model is well suited to capture the risk attitude of the agents in the model economy that can be inferred from asset prices data.

### 4.3 The Great Moderation in the Asset Pricing Model

As just showed, the proposed calibration for the model does remarkably well on matching the first and second moments of asset prices during the whole sample. Therefore, I will fix the preference

parameters to their calibrated values through the rest of the paper. Obviously, it is crucial that preferences parameters are held constant when doing welfare comparison between two different specifications of the model: the ability of the model to capture the overall behavior of asset pricing guarantees that the preference parameters are at least reasonable.

The next step, then, is to model the Pre-Great Moderation and the Great Moderation period. As suggested in the first section of this paper, the properties of the consumption and dividend time-series have changed during the Great Moderation both in their variance and persistence. In this section I will calculate the welfare impact of these changes. However, there is also an additional component that might have changed, which is the magnitude of the long-run risk shock. Since, as stated, this component is hardly identifiable in the data, I will try to infer it from the asset pricing data.

#### **4.3.1 Sample 1 Calibration: the pre-Great Moderation**

First I consider the pre-Great Moderation period, 1947:1-1982:4. Fixing the preference parameter as in the previous sub-section (i.e.  $\gamma = 12$ ,  $\delta = 0.998$ ,  $\Psi = 1.5$ ), I adjust the autoregressive parameters of the consumption and dividend growth process and the standard deviation of their innovations to their estimated values for Sample 1 as reported in Table 2. I also adjust the values of the constants  $\mu^c$  and  $\mu^d$  in the equations (6) and (7) to imply an average growth rate of consumption and dividend equal to the one in the whole sample: keeping the mean growth rate of consumption and dividends constant across the pre-Great Moderation and Great Moderation samples assures that the welfare calculations are completely driven by second moment effects (changes in the variance and persistence of dividends and consumption) and not by first moment effects (changes in the average level of dividends and consumption). This assumption is supported by the remarkable stability of the mean consumption and dividend growth in the two sub-sample as showed in Table 1. Finally, I adjust the level of the standard deviation of the long-run risk shock  $\sigma_x$  to match the Sharpe Ratio in the Sample 1. The second column of Table 6 displays the resulting calibrated parameters for Sample 1. The only important feature to be noticed is that to match the Sharpe Ratio in the pre-Great Moderation, the relative variance of the long-run

innovation with respect to the consumption variance innovation,  $\sigma_x$ , is slightly lower than the one for the whole sample.

What are the asset pricing implications of the model for the pre-Great Moderation period? The left panel of Table 8 shows that the model is overall able to match the first and second moments of financial variables in the pre-Great Moderation period; only the average risk-free rate is slightly above its data counterpart, and the model implied risk-free rate volatility is below the one observed in the data.

#### 4.3.2 Sample 2 Calibration: the Great Moderation

I then repeat the same procedure for the Great Moderation period. First I adjust the autoregressive parameters of the consumption and dividend processes and the standard deviation of their innovations to their estimated value for the sample 1983:1-2007:4. Then I adjust the constants  $\mu^c$  and  $\mu^d$  to imply an average dividend and consumption growth exactly equal to the one of the pre-Great Moderation period. Next, I adjust the level of the standard deviation of the long-run risk shock to match the Sharpe Ratio in the Great Moderation period. I also vary the parameter that links the dividend and consumption process,  $\lambda_d$ , from 2.5 to 2.1 to match the relative variance of dividends with respect to the one of consumption growth, and it is consistent with a lower correlation between dividend and consumption growth rates estimated during the Great Moderation period. The third column of Table 6 displays the resulting calibration. Again, the only important feature of the calibration is the slightly higher value of  $\sigma_x$  in the Sample 2 with respect to the one calibrated for the Sample 1. Notice that this value does not imply a larger variance of the long-run risk shock in the Great Moderation period, but only its larger relative variance with respect to the consumption innovation variance<sup>17</sup>.

The right panel of Table 8 displays the asset pricing implications derived from the long-run risk model when calibrated for the Great Moderation period. Overall, the model matches the asset pricing data very closely, except for the too low volatility of the risk-free rate.

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<sup>17</sup>The standard deviation of the long-run risk shock is, in fact, given by  $\sigma_x \sigma_c$ , which is equal to 0.018 percent in Sample 1 and 0.012 percent in Sample 2.

In conclusion, the calibrated preference parameters and long-run risk process parameters, together with the estimated laws of motion of consumption and dividend growth, deliver model asset pricing implications that successfully match their data counterpart in both the pre-Great Moderation and Great Moderation periods.

## 4.4 Counterfactual Scenarios

As showed in Table 5, the calibration considered for the Great Moderation period differs from the one for the pre-Great Moderation period in three dimensions. First, there is a change in the variances of the innovation of the growth rate of consumption and dividend. I will label this effect as “innovation-effect”. Second, there is a change in the autocorrelation structure of the consumption and dividend growth process. I will label this effect as “persistence-effect”. Third, there is a change in relative magnitude of the long-run shock with respect to the consumption shock. I will name this effect as “long-run-risk-effect”. The empirical analysis in Section 2 suggests that: (1) the “innovation-effect” has contributed to reduce the variance of macroeconomic variables during the Great Moderation, and (2) the “persistence-effect” has increased their persistence. In addition, the model suggests that, in order to match the asset pricing data in the pre- and during-Great Moderation period, a positive “long-run-risk-effect” is needed, that is a higher relative long-run risk variance with respect to consumption variance,  $\sigma_x$ . In this section I propose some counterfactual scenarios that are helpful on isolating the welfare implications of each of these effects.

The “innovation-effect” can be isolated considering a scenario where all the autoregressive parameters of the consumption and dividend process, as well as the relative magnitude of the long-run risk shock, are held constant to the pre-Great Moderation period, and the standard deviation of consumption and dividends is scaled down to match the unconditional variance of consumption and dividend growth in Sample 2. Intuitively, this scenario captures a counterfactual Great Moderation characterized only by a smaller magnitude of consumption and dividends shocks that matches the same variance of consumption and dividends as the one observed in the Great Moderation period.

The “persistence-effect” can be isolated by assuming that a counterfactual Great Moderation is

solely characterized by a change of the autoregressive parameters  $[\theta_1^c, \theta_2^c, \theta_3^c]$  and  $[\theta_1^d, \theta_2^d, \theta_3^d]$ . In this scenario, the variance of the consumption and dividend innovations is adjusted to imply the same unconditional variance of consumption and dividend as in Sample 1. Also the long-run risk variance is held constant to the Sample 1 variance. Intuitively, this scenario captures a counterfactual Great Moderation characterized only by a change in the autocorrelation structure of consumption and dividend that delivers the same variance of consumption and dividends as the one observed in the Great Moderation period.

Finally, the “long-run-risk effect” captures a counterfactual scenario where all the variances and autoregressive coefficients of dividends and consumption are held fixed at their Sample 1 values and the relative magnitude of the long run risk shock is equal to its Sample 2 value, 0.04.

The last three columns of Table 6 describe rigorously the parameterization of the three scenarios.

## 5 The Welfare Gain or Loss of the Great Moderation

What is the welfare gain or loss brought by the Great Moderation? I answer this question by using two alternative methods. First, I consider the model-free approach suggested by Alvarez and Jermann (2004). This approach extracts information on welfare from consumption data and asset pricing data. The appeal of this method stems from not relying in a specific general equilibrium model. Unfortunately, for the same reason, this approach does not obviously allow to isolate welfare effects generated by different sources through counterfactual scenarios. In order to understand in depth welfare gains and losses during the Great Moderation period, I will then use the asset pricing model presented in the previous section to compute the welfare gain/loss brought by the Great Moderation and I will quantify the magnitude of the three effects presented above: the “innovation-effect”, the “persistence-effect”, and the “long-run risk-effect”.

## 5.1 The Model-Free Welfare Calculation

Alvarez and Jermann (2004) proposed a method to compute the marginal cost of consumption fluctuations  $\omega_0$  as a function of an interest rate, the average growth rate of consumption, and a consumption risk premium, under the assumption that consumption growth is independently and identically distributed, and the interest rate is constant. Whereas, the interest rate and consumption growth rates are observable in the data, the consumption risk premium is estimated from the returns of a reference portfolio. In detail, using the Alvarez and Jermann (2004)'s notation, the marginal cost of all uncertainty is:

$$\omega_0 = \frac{y + p - g}{y - g},$$

where  $y$  is a constant long-term bond interest rate,  $p$  is the estimated mean consumption risk premium,  $g$  is the average growth rate of consumption. It is straightforward to compute the welfare gain of eliminating all uncertainty in the pre-Great Moderation and Great Moderation period. For the pre-Great Moderation period, I set  $y = 3.02$  percent (annual), which is the real return of 10-year Treasury bond in Sample 1; I set  $g = 1.96$  percent (annual), which is the average growth rate in Sample 1; finally,  $p$  is estimated by using a Fama-French portfolio of returns for the period 1947:1-1982:4<sup>18</sup>. The resulting average value of the consumption risk is  $p = 0.15$  percent. Accordingly, the welfare gain of eliminating all fluctuation in Sample 1 is 0.15 percent. Similarly, for the Great Moderation sub-sample, the average long-run interest rate is  $y = 3.75$  percent (annual), the average growth rate of consumption is equal to the one in Sample 1, i.e.  $g = 1.96$  percent (annual), and the estimated mean consumption risk premium is  $p = 0.075$  percent. Consequently, the resulting welfare gain of eliminating all fluctuation in Sample 2 is 0.04 percent. The welfare gain of the Great Moderation is computed by calculating the difference between the welfare gain of eliminating all fluctuation in Sample 1 and the one in Sample 2: therefore, the welfare gain of the Great Moderation is equal to 0.11 percent.

Therefore, the model free welfare calculation that takes into account consumption and asset pricing data attributes a relatively small gain brought by the Great Moderation.

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<sup>18</sup>Source: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

## 5.2 The Model-Based Welfare Calculations

The model-free approach presented above is not able to isolate the possible different effects that have contributed to a lower welfare gain of the Great Moderation, since its data-based method does not allow to run counterfactual scenarios. In this section, I use the long-run risk model presented above to compute the welfare gain/loss of the Great Moderation, and to study the welfare contribution of the three effects characterizing the Great Moderation period: the “innovation-effect”, the “persistence-effect”, and the “long-run risk effect”. The proposed model is suitable for welfare calculation since it can successfully replicate the asset pricing moments, as well as the dynamics of consumption and dividends, in the pre-Great Moderation and Great Moderation period, as previously discussed.

The welfare computation for the long-run risk model follows the approach proposed by Croce (2012). Let  $U(\{C^j\})$  be the time-zero utility obtained from the consumption process  $\{C^j\}$  and  $u^j$  the associated logarithm of the utility-consumption ratio. The percentage change of time-zero consumption,  $\lambda$ , that one must give to an agent in order to render her indifferent between the consumption process  $\{C^j\}$  and an alternative consumption process  $\{C^i\}$  is given by:

$$\lambda = u^j - u^i.$$

Given the Epstein-Zin preferences, the log-utility-consumption ratio satisfies:

$$u_t = \log \left( [(1 - \delta)(v_{c,t} + 1)]^{\frac{1}{1-\Psi}} \right).$$

Therefore, given that the model provides a solution for the dynamics of the price-consumption ratio,  $v_{c,t}$ , we can also compute the dynamics of the utility-consumption ratio. Finally, I define the welfare change from moving from the pre-Great Moderation specification of the consumption (and dividends) process to different alternative specifications as the model implied average of the associated utility-consumption ratios.

What is the welfare change, implied by the model, brought by the Great Moderation? The first column of Table 9 displays the result: there is a welfare loss associated with the Great Moderation,



and it is equal to -0.9 percent of in consumption equivalent terms. This result might be surprising at first, but in order to understand why an agent is not willing to move from the pre-Great Moderation to the Great-Moderation consumption (and dividends) process, we need to isolate the three effects that characterized the Great Moderation specification, namely a lower variance of the innovation of consumption and dividends (innovation-effect), an increased persistence of the consumption and dividends process (persistence effect), and an increased relative variance of the long-run risk component (long-run risk shock).

Using the counterfactual scenario labeled as “innovation-effect” in Table 9, we observe that the reduction of the variance of the innovation alone is welfare improving: an agent would be willing to move from the Sample 1 process of consumption and dividend to an alternative process with the same autoregressive structure and a constant relative long-run risk variance, but with a lower variance of consumption and dividend innovations. The agent would be willing to give up 1.6 percent of her consumption for that. This is quite intuitively, since agents are risk adverse and, *ceteris paribus*, they highly value the lower consumption variance. Notice that in this scenario, the mean equity premium implied by the model would drastically decline to 2.4 percent.

However, the reduction of the innovation variance of consumption and dividend is not the only feature of the Great Moderation. The empirical analysis in Section 2, shows that Great Moderation is also characterized by a larger persistence of consumption and dividend. The counterfactual scenario labeled as “persistence effect” in Table 9 captures a pure shift on the autoregressive structure of consumption and dividend without altering their unconditional variance and the long-run risk shock variance. When the persistence of consumption and dividend increases, agents’ welfare is largely reduced (-6.6 percent), and the mean equity premium would jump to a 25 percent. Intuitively, an agent endowed with Epstein-Zin preferences with intertemporal elasticity of substitution greater than 1 is adverse to postpone uncertainty resolution. When the persistence of consumption and dividend increases, their volatility shifts from the short-run to the long-run, thus making the agent worse off. As a result, the agent would ask for a larger premium to hold a risky asset (25.6 percent). Figure 1 visualizes the relationship between the estimated consumption growth persistence (measured as the highest root of the third order lag polynomial) in the Great

Moderation period, and the welfare gain/loss brought by the Great Moderation for the 1000 bootstrap replications used to compute the 90 percent confidence band of the welfare change. It is evident the negative relation between welfare gain of the moderation and the persistence of the Great Moderation consumption process.

Finally, the Great Moderation calibration features also a slightly higher variance of the long-run risk components relative to the consumption variance. In the counterfactual scenario labeled “long-run-risk-effect”, the variance and persistence of consumption and dividends is held fixed to the Sample 1 values, and the  $\sigma_x$  is increased to its Sample 2 value, 0.04. Since the higher relative increased variance of the long run risk induces also a larger long-run uncertainty, the agent assigns a negative welfare effect to it (-1.4 percent). Also in this case, he would ask for a larger equity premium to hold risky asset (12 percent).

The main result of the paper is summarized as follow: the gain brought of the Great Moderation is rather small, if not absent at all. The reduction of the magnitude of the consumption and dividend innovation shock implies an unambiguous welfare gain. But the Great Moderation period was not solely characterized by a reduction of consumption and dividends variance. The data, in fact, show that the Great Moderation also features a larger portion of consumption and dividend variance concentrated in the long-run, caused by their increased persistence and by the larger relative long-run-risk. These two properties are welfare deteriorating, since they delay the resolution of uncertainty from the investor point of view.

## 6 Alternative Models

In this section I explore whether my estimated small gain brought on by the Great Moderation is robust to alternative modelling choices: I analyze the predictions of three alternative models that are successful in solving the equity premium puzzle: the habit model of Campbell and Cochrane (1999), and the rare disaster model of Barro (2006, 2009).

## 6.1 Campbell and Cochrane Model

Another model that is able to match the asset pricing moments is the habit model introduced by Campbell and Cochrane (1999). However, one crucial limitation of this model for computing welfare analysis is the implied relationship between preference parameters and the parameters of the exogenous laws of motion. In conducting welfare calculations we generally want to hold preference parameters fixed, while experimenting with the law of motion of consumption. This is, unfortunately, impossible with Campbell and Cochrane's model.

There are three important features of the Campbell-Cochrane model. First, external habit formation in the utility function, second, a slow response of habit to consumption, and third, a non-linear relationship between habit and consumption. In particular, the agent's instantaneous utility is  $U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}$ , where  $C_t$  is the level of consumption, and  $X_t$  is the stock of habit. Define the surplus consumption ratio,  $S_t$  as

$$S_t = \frac{C_t - X_t}{C_t}.$$

The law of motion of the habit stock is modelled specifying a heteroskedastic AR(1) process for the log surplus consumption ratio,  $s_t$ , i.e.

$$s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) (c_{t+1} - c_t - g),$$

where  $\bar{s}$ ,  $g$ , and  $\phi$  are parameters, and  $c_{t+1} - c_t - g = v_{t+1}$  is an i.i.d. normal process with mean 0 and standard deviation  $\sigma$ . The function  $\lambda(s_t)$  is the *sensitivity function* specified as follows

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{if } s_t > s_{\max} \end{cases},$$

with  $s_{\max} = \bar{s} + \frac{1}{2} (1 - \bar{S}^2)$ . The parameter  $\bar{S}$  is the steady state surplus consumption ratio and is

defined as follows, imposing some useful conditions on the sensitivity function<sup>19</sup>:

$$\bar{S} = \sigma \sqrt{\frac{\gamma}{1 - \phi}}.$$

The sensitivity function measures the response of the surplus consumption ratio to innovations in consumption growth. Notice that since  $\bar{S}$  is proportional to  $\sigma$ , a less volatile consumption growth process, such as that experienced in the Great Moderation, implies a lower steady-state surplus consumption ratio. Moreover, the functional form of the sensitivity function indicates that a less volatile consumption growth process is associated with higher values of the sensitivity function, holding  $\gamma$  and  $\phi$  constant. As a result, fixing the percentage deviation of the log-surplus consumption ratio from its steady state,  $s_t - s$ , the distribution of  $s_t$  associated with a less volatile consumption growth process shifts to the left and does not change its variance, since the lower volatility of the  $v_t$  process is amplified by a larger sensitivity function.

Although this mechanism helps to reconcile the model predictions with several otherwise puzzling asset pricing data moments, namely, the average risk-free return and the average equity premium as well as their volatilities, and the Sharpe ratio of equity returns, it also creates some counter-intuitive welfare implications. In fact, a reduction of the volatility of consumption growth leads to a decline of the surplus consumption ratio, which is the variable from which the agent gains utility. Thus, a less volatile growth rate of consumption has a negative effect on utility.

In the previous section I showed that the volatility of consumption growth has declined from 0.62 percent in the pre-1983 sample to 0.34 percent in the post-1983 sample. When these estimates are applied to the Campbell and Cochrane model, the welfare loss implied by the less volatile consumption process is 5 percent in consumption-equivalent terms. This result seems paradoxical since we usually expect that a risk-averse agent would prefer a smoother consumption process. However, the left-ward shift on the distribution of the surplus consumption ratio is equivalent to a change in the preference parameters of the agent, or, in other words, to a re-scaling of the variable from which the agent gains utility. Therefore the decline in utility implied by this calculation is mainly due to the cardinal value of the utility function, which has no meaning in terms of welfare.

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<sup>19</sup>See Campbell and Cochrane (1999), p.213.

## 6.2 Rare Disaster Model

In the rare disaster model the equity premium is generated by two components; the first one is proportional to the variance of consumption growth, and the second one depends on the probability and magnitude of the rare disaster. In particular, the expression for the equity premium is

$$R^{RP} = \gamma\sigma^2 + pE\left(b\left[(1-b)^{-\gamma} - 1\right]\right),$$

where  $\gamma$  is the coefficient of relative risk aversion,  $\sigma^2$  is the variance of consumption growth,  $p$  is the probability that a disaster occurs, and  $b$  is the magnitude of the disaster. Using Barro's calibration, the risk-premium implied by the model is 5.9 percent. However, only 0.16 percent is due to the first component. Therefore, the impact of the volatility of consumption growth is negligible if compared to the contribution of the rare disaster. This observation suggests that a reduction in the volatility of consumption growth will have a small impact on welfare calculations based on this model. In fact, assuming that the probability and the magnitude of a disaster did not change in the Great Moderation, a 50 percent decline of the standard deviation of consumption growth, as experienced in the post-1983 sample, implies a welfare gain of 0.84 percent. Since the agent in this model is mainly concerned about disaster risk, and this is what is being priced in the equity markets, a change in day-to-day "normal" volatility has only a limited effect on welfare.

## 7 Conclusions

In this paper I estimated the welfare improvement brought about by the Great Moderation, the reduction in the business cycle volatility of macroeconomic variables after the early 1980s. Using simple consumption-based asset pricing models, I showed that the welfare estimates and the moments of asset prices are very sensitive to the time-series properties of the consumption processes that are fed into these calculations.

The contribution of this paper is to take very seriously the need for welfare calculations to be based on plausibly calibrated laws of motion of consumption (and dividends), and on models which have reasonable predictions for asset prices. I document that the reduction in volatility

of consumption and dividend growth rate in the Great Moderation period is associated with an increase of their persistence. Therefore, I develop an asset pricing model (long-run risk model) in which the laws of motion of consumption and dividends capture their changing autocorrelation structure. With a set of calibrated preference parameters, the proposed model delivers sensible asset price behavior over the full sample and in the two sub-samples. I then compute that the Great Moderation has brought a welfare loss equal to -0.9 percent in consumption-equivalent terms. I build three counterfactual scenarios that isolate the effects of the Great Moderation: whereas the smaller magnitude of the consumption-dividend shock is welfare improving, the increased persistence of the two process is severely welfare deteriorating. In fact, a risk-averse agent endowed with non-separable preference characterized by an elasticity of substitution larger than one prefers an early resolution of uncertainty with respect to a late one. The increased persistence of the two exogenous process, then, contribute to the welfare loss. By considering a model-free approach, as well as two other alternative models, I conclude that there is no evidence that the Great Moderation was significantly welfare improving.

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TABLE 1: DESCRIPTIVE STATISTICS OF CONSUMPTION AND DIVIDENDS

|                                  | All Sample     | Sample 1       | Sample 2       |
|----------------------------------|----------------|----------------|----------------|
|                                  | 1947Q1-2007Q4  | 1947Q1-1982Q4  | 1983Q1-2007Q4  |
| $\mu(\Delta(c_t))$               | 0.49<br>[0.04] | 0.49<br>[0.06] | 0.48<br>[0.06] |
| $\mu(\Delta(d_t))$               | 0.58<br>[0.24] | 0.54<br>[0.35] | 0.63<br>[0.29] |
| $\sigma(\Delta(c_t))$            | 0.53<br>[0.05] | 0.62<br>[0.05] | 0.34<br>[0.04] |
| $\sigma(\Delta(d_t))$            | 1.78<br>[0.30] | 2.08<br>[0.40] | 1.20<br>[0.18] |
| $\rho(\Delta(d_t), \Delta(c_t))$ | 0.07<br>[0.07] | 0.10<br>[0.09] | 0.05<br>[0.09] |

Note: This table displays the descriptive statistics of consumption growth rate,  $\Delta(c_t)$ , and dividend growth rate,  $\Delta(d_t)$ , in the whole sample, pre-Great Moderation period (Sample 1) and Great Moderation period (Sample 2) (mean,  $\mu(\cdot)$ , in percent, standard deviation,  $\sigma(\cdot)$ , in percent, and correlation,  $\rho(\cdot, \cdot)$ ). Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

TABLE 2: ESTIMATED AR(3) PARAMETERS FOR CONSUMPTION AND DIVIDEND GROWTH

|            | Whole Sample     |                    | Sample 1         |                    | Sample 2          |                   |
|------------|------------------|--------------------|------------------|--------------------|-------------------|-------------------|
|            | 1947Q1-2007Q4    |                    | 1947Q1-1982Q4    |                    | 1983Q1-2007Q4     |                   |
|            | $\Delta(c_t)$    | $\Delta(d_t)$      | $\Delta(c_t)$    | $\Delta(d_t)$      | $\Delta(c_t)$     | $\Delta(d_t)$     |
| $\theta_1$ | 0.12*<br>[0.06]  | 0.74***<br>[0.06]  | 0.10<br>[0.08]   | 0.75***<br>[0.06]  | 0.21**<br>[0.06]  | 0.59***<br>[0.06] |
| $\theta_2$ | 0.16**<br>[0.06] | -0.24***<br>[0.07] | 0.17**<br>[0.08] | -0.30***<br>[0.06] | 0.08<br>[0.06]    | -0.02<br>[0.06]   |
| $\theta_3$ | 0.10**<br>[0.06] | 0.16**<br>[0.06]   | 0.06<br>[0.08]   | 0.13<br>[0.06]     | 0.32***<br>[0.06] | 0.26**<br>[0.06]  |

Note: This table reports the estimated autoregressive parameters for consumption growth rate,  $\Delta(c_t)$ , and dividend growth rate,  $\Delta(d_t)$ , in the whole sample, pre-Great Moderation period (Sample 1) and Great Moderation period (Sample 2). The econometric model considered is as in (1). Standard errors are reported in brackets. Statistical significance is indicated with asterisks (\* at 10 percent, \*\* at 5 percent, \*\*\* at 1 percent).

TABLE 3: ESTIMATED PERSISTENCE AND INNOVATION VARIANCE: CONSUMPTION AND  
DIVIDEND GROWTH

|                   | Whole Sample          |                       | Sample 1              |                       | Sample 2              |                       |
|-------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
|                   | 1947Q1-2007Q4         |                       | 1947Q1-1982Q4         |                       | 1983Q1-2007Q4         |                       |
|                   | $\Delta(c_t)$         | $\Delta(d_t)$         | $\Delta(c_t)$         | $\Delta(d_t)$         | $\Delta(c_t)$         | $\Delta(d_t)$         |
| $\rho$            | 0.64<br>[0.44 ; 0.71] | 0.72<br>[0.54 ; 0.78] | 0.58<br>[0.31 ; 0.68] | 0.60<br>[0.10 ; 0.70] | 0.81<br>[0.60 ; 0.86] | 0.90<br>[0.63 ; 0.95] |
| $\sigma_\epsilon$ | 0.49<br>[0.42 ; 0.52] | 1.00<br>[0.96 ; 1.27] | 0.56<br>[0.49 ; 0.51] | 1.27<br>[1.02 ; 1.43] | 0.29<br>[0.23 ; 0.33] | 0.82<br>[0.65 ; 0.98] |

Note: This table reports the estimated persistence and innovation standard deviation (percent) for consumption growth rate,  $\Delta(c_t)$ , and dividend growth rate,  $\Delta(d_t)$ , in the whole sample, pre-Great Moderation period (Sample 1) and Great Moderation period (Sample 2). The persistence parameter is defined as the largest root of the third order autoregressive polynomial estimated as in (1). In brackets I report the 90 percent confidence band obtained via bootstrap (10000 replications).

TABLE 4: MOMENTS OF ASSET PRICES

| Annualized Mean Return and Standard Deviations (Percent) |                 |                 |                 |
|--|-----------------|-----------------|-----------------|
|  | Whole Sample    | Sample 1        | Sample 2        |
|  | 1947Q1-2007Q4   | 1947Q1-1982Q4   | 1983Q1-2007Q4   |
| Equity Return: Mean                                      | 8.68<br>[1.76]  | 7.74<br>[2.43]  | 10.00<br>[2.64] |
| Risk Free Asset Return: Mean                             | 0.98<br>[0.45]  | 0.33<br>[0.49]  | 1.88<br>[0.60]  |
| Risk Premium: Mean                                       | 7.70<br>[1.68]  | 7.42<br>[2.48]  | 8.11<br>[2.33]  |
| Equity Return: Standard Deviation                        | 15.99<br>[1.20] | 15.90<br>[1.54] | 16.07<br>[1.90] |
| Risk Free Asset Return: Standard Deviation               | 1.29<br>[0.18]  | 1.35<br>[0.25]  | 1.07<br>[1.92]  |
| Risk Premium: Standard Deviation                         | 15.91<br>[1.17] | 15.91<br>[1.53] | 16.07<br>[0.16] |
| Sharp Ratio  | 0.48            | 0.46            | 0.50            |

Note: This table reports mean and standard deviation of financial variables in the whole sample, pre-Great Moderation period (Sample 1) and Great Moderation period (Sample 2). Heteroskedasticity consistent standard errors computed with the Newey-West procedure in brackets.

TABLE 5: ASSET RETURNS AND WELFARE IMPLICATIONS OF A TIME-SEPARABLE MODEL WITH ALTERNATIVE SPECIFICATIONS OF THE CONSUMPTION PROCESS

|  | Linear trend + iid  | iid Consumption Growth | AR(1) Consumption Growth |
|--|---|------------------------|--------------------------|
| Discount factor ( $\beta$ )                | 0.95  | 0.95                   | 0.95                     |
| Risk aversion ( $\gamma$ )                 | 2   | 2                      | 2                        |
|  |   |                        |                          |
|  | Annualized Mean Return (Percent)                              |                        |                          |
| Equity Return                              | 11.8  | 11.8                   | 9.1                      |
| Risk Free Asset Return                     | 11.7  | 11.8                   | 8.8                      |
| Risk Premium                               | 0.1   | 0.0                    | 0.3                      |
|  | Welfare change in terms of consumption compensation (Percent) |                        |                          |
| Welfare gain from eliminating fluctuations | 0.02  | 0.1                    | 0.65                     |
|  |   |                        |                          |
|  |   |                        | 7.4                      |

Note: The model assumes a CRRA utility function. The “linear trend +iid” consumption process and the utility parameters in the second column are as in Lucas (1987). The “iid Consumption Growth” process in the third column is calibrated to match the post war data. The  $AR(1)$  consumption growth process and utility parameters in the fourth column are as in Mehra and Prescott (1985). The  $AR(1)$  consumption growth process in the fifth column has the same specification as in Mehra and Prescott (1985). However, the utility parameters are calibrated to match a 6 percent risk premium and a 1.4 percent risk free asset return

TABLE 6: CALIBRATION

| Whole Sample                           | Sample 1              | Sample 2              | Innovation Effect     | Persistence Effect    | LLR Effect            |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1947Q1-2007Q4                          | 1947Q1-1982Q4         | 1983Q1-2007Q4         |                       |                       |                       |
| Consumption Growth Parameters          |                       |                       |                       |                       |                       |
| $[\theta_1^c; \theta_2^c; \theta_3^c]$ | $[0.16; 0.12; 0.10]$  | $[0.19; 0.10; 0.32]$  | $[0.14; 0.11; 0.01]$  | $[0.19; 0.10; 0.32]$  | $[0.19; 0.10; 0.32]$  |
| $\mu^c$                                | 0.003                 | 0.001                 | 0.003                 | 0.001                 | 0.001                 |
| $\sigma^c$                             | 0.005                 | 0.003                 | 0.003                 | 0.005                 | 0.003                 |
| Dividend Growth Parameters             |                       |                       |                       |                       |                       |
| $[\theta_1^d; \theta_2^d; \theta_3^d]$ | $[0.76; -0.32; 0.30]$ | $[0.58; -0.01; 0.26]$ | $[0.79; -0.39; 0.31]$ | $[0.58; -0.01; 0.26]$ | $[0.58; -0.01; 0.26]$ |
| $\mu^d$                                | 0.001                 | 0.001                 | 0.001                 | 0.001                 | 0.001                 |
| $\sigma^d$                             | 0.012                 | 0.008                 | 0.008                 | 0.012                 | 0.008                 |
| $\lambda_d$                            | 2.5                   | 2.1                   | 2.5                   | 2.5                   | 2.1                   |
| Long Run Risk Parameters               |                       |                       |                       |                       |                       |
| $\rho_x$                               |                       |                       | 0.98                  |                       |                       |
| $\sigma_x$                             | 0.032                 | 0.031                 | 0.04                  | 0.031                 | 0.04                  |
| Preference Parameters                  |                       |                       |                       |                       |                       |
| $\delta$                               |                       |                       | 0.998                 |                       |                       |
| $\gamma$                               |                       |                       | 12                    |                       |                       |
| $\Psi$                                 |                       |                       | 1.5                   |                       |                       |

Note: This table reports the calibration used for the long-run risk model. Recall that the adjustment of  $\sigma_c$  and  $\sigma_{ma_d}$  in the persistence-effect scenario is needed to control for a constant unconditional variance of consumption and dividend growth, since a change in the autocorrelation structures implies a change in the unconditional variance of the process.

TABLE 7: MODEL IMPLICATIONS: WHOLE SAMPLE

|                                 | Model | Data  |
|---------------------------------|-------|-------|
| Consumption and Dividend Growth |       |       |
| Mean Consumption                | 0.49  | 0.49  |
| Std.Dev. Consumption            | 0.55  | 0.54  |
| Mean Dividend                   | 0.57  | 0.58  |
| Std.Dev. Dividend               | 1.80  | 1.78  |
| Correlation Cons-Div.           | 0.15  | 0.07  |
| Asset Pricing                   |       |       |
| Mean Equity Premium             | 7.73  | 7.70  |
| Mean Risk-Free Rate             | 1.85  | 0.98  |
| Mean Equity Rate                | 9.58  | 8.68  |
| Std.Dev. Equity Premium         | 15.79 | 15.91 |
| Std.Dev. Risk-Free Rate         | 0.30  | 1.29  |
| Std.Dev. Equity Return          | 15.77 | 15.99 |
| Sharpe Ratio                    | 0.48  | 0.48  |

Note: This table compares the model and data moments of consumption and dividend growth (top panel) and asset pricing variables (bottom panel) as implied by the long-run risk model calibrated for the whole sample (1947:1-2007:4), as well as their data counterpart. Means and standard deviations are reported in percent.

TABLE 8: MODEL IMPLICATIONS SUB-SAMPLE AND COUNTERFACTUAL

|                         | Sample 1 |       | Sample 2 |       |
|-------------------------|----------|-------|----------|-------|
|                         | Model    | Data  | Model    | Data  |
| Mean Equity Premium     | 7.37     | 7.42  | 8.00     | 8.12  |
| Mean Risk-Free Rate     | 1.8      | 0.33  | 1.83     | 1.88  |
| Mean Equity Rate        | 9.23     | 7.74  | 9.82     | 10.00 |
| Std.Dev. Equity Premium | 16.09    | 15.91 | 15.78    | 16.07 |
| Std.Dev. Risk-Free Rate | 0.26     | 1.35  | 0.32     | 1.07  |
| Std.Dev. Equity Return  | 16.07    | 15.90 | 15.77    | 16.17 |
| Sharpe Ratio            | 0.46     | 0.46  | 0.50     | 0.50  |

Note: This table compares the model and data moments of consumption and dividend growth (top panel) and asset pricing variables (bottom panel) as implied by the long-run risk model calibrated for the pre-Great Moderation period (Sample 1, 1947:1-1982:4) and for the Great Moderation period (Sample 2, 1983:1-2007:4), as well as their data counterpart. Means and standard deviations are reported in percent.

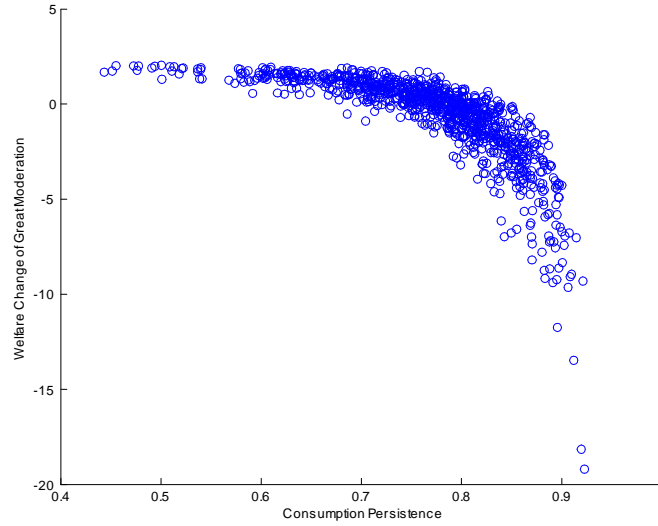


TABLE 9: WELFARE CHANGE WITH RESPECT TO THE PRE-GREAT MODERATION PERIOD

|                              | (Percent)             |                     |              |
|------------------------------|-----------------------|---------------------|--------------|
|                              | Welfare Change        | Mean Equity Premium | Sharpe Ratio |
| Sample 1<br>Great Moderation | -0.9<br>[-4.8 ; 1.4 ] | 8.00                | 0.50         |
| Innovation Effect            | 1.6                   | 2.41                | 0.25         |
| Persistence Effect           | -6.6                  | 25.6                | 0.15         |
| Long-Run Risk Effect         | -1.4                  | 12.8                | 0.24         |

Note: This table reports the welfare change implied by alternative calibration of the model with respect to the pre-Great Moderation period (1947:1-1982:4), as well as the mean equity premium and the Sharpe ratio implied by each calibration. The first row, displays the welfare change from the Great Moderation period (90 percent confidence band computed via bootstrap in brackets). The bottom panel reports the welfare change implied by the three counterfactual scenario. The innovation effect is characterized by only a reduction of the variance of consumption and dividend innovation. The Persistence effect is characterized by only a change in the autocorrelation structure of consumption and dividend (keeping the unconditional variance constant to the Sample 1 values). The Long-Run Risk effect is characterized only by an increased in the relative variance of the long-run risk process with respect to the consumption variance.

FIGURE 1: WELFARE GAIN AND CONSUMPTION PERSISTENCE



Note: This figure displays the relationship between the estimated Sample 2 persistence of the consumption process in each of the 1000 bootstrap iteration (x-axis) and the welfare change of moving from the Sample 1 process to the bootstrapped Sample 2 process. Persistence is measured as the largest root of the third order lag polynomial. Welfare changed is defined in consumption equivalent terms.

## 8 Appendix A

This Appendix describes the Lucas (1978) endowment economy. The representative agent maximizes the lifetime expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where  $E_0$  denotes the conditional expectations given the information at time 0,  $U(\cdot)$  denotes the instantaneous utility function,  $C_t$  denotes consumption at time  $t$ , and  $\beta$  is the discount factor. There is a competitive market for trading assets (trees) which pay dividends (fruits). Let  $P_t$  be the price of one unit of the asset and  $A_t$  be the agent's shareholding at time  $t$ , then the agent's budget constraint is

$$C_t + P_t A_{t+1} = (P_t + D_t) A_t,$$

where  $d_t$  denotes the exogenous stochastic flow of fruits at time  $t$ . Since there is no source of the consumption good other than the fruit, which is perishable, market clearing implies that  $C_t = D_t$ .

As in Lucas, the agent has CRRA preferences, i.e.

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma},$$

where  $\gamma > 1$  is the coefficient of risk aversion. Since  $\gamma$  is positive, the agent in the economy is risk-averse.

The price of the asset is determined by the first order conditions as follows:

$$P_t = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (P_{t+1} + C_{t+1}) \right]. \quad (9)$$

To link the asset pricing variables and the welfare cost of fluctuations, it is useful to rewrite (9) in terms of the price-dividend ratio  $V_t = P_t/D_t$ :

$$V_t = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} (V_{t+1} + 1) \right]. \quad (10)$$